Division A

Coordinate System Issues in Binary Star Computations

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It has been estimated that half of all stars are components of binary or multiple systems. Yet the number of known orbits for astrometric and spectroscopic binary systems together is less than 7,000 (including redundancies), almost all of them for bright stars. A new generation of deep all-sky surveys such as Pan-STARRS, Gaia, and LSST are expected to lead to the discovery of millions of new systems. Although for many of these systems, the orbits may be poorly sampled initially, it is to be expected that combinations of new and old data sources will eventually lead to many more orbits being known. As a result, a revolution in the scientific understanding of these systems may be upon us.

The current database of visual (astrometric) binary orbits represents them relative to the "plane of the sky", that is, the plane orthogonal to the line of sight. Although the line of sight to stars constantly changes due to proper motion, aberration, and other effects, there is no agreed upon standard for what line of sight defines the orbital reference plane. Furthermore, the computation of differential coordinates (component B relative to A) for a given date must be based on the binary system's direction at that date. Thus, a different "plane of the sky" is appropriate for each such date, i.e., each observation. However, projection effects between the reference planes, differential aberration, and the curvature of the sky are generally neglected in such computations. Usually the only correction applied is for the change in the north direction (position angle zero) due to precession (and sometimes also proper motion). This paper will present an algorithm for a more complete model of the geometry involved, and will show that such a model is necessary to avoid errors in the computed observables that are significant at modern astrometric accuracy. The paper will also suggest where conventions need to be established to avoid ambiguities in how quantities related to binary star orbits are interpreted.

Motivation

This investigation began as a relatively simple project to add a binary-star software module to the Naval Observatory Vector Astrometry Software (NOVAS). NOVAS it is a free source-code library for high-precision astrometric calculations available at http://aa.usno.navy.mil/software/novas/ in Fortran, C, and Python. It is likely that as observational accuracy increases, binary stars will become an increasingly important issue in defining celestial reference frames. For example, even when observations span only a few years, the components of wide pairs with orbital periods of centuries will show detectable curvatures of their paths if the observational accuracy is great enough (Kaplan & Makarov 2004, AN 324, 419; Makarov & Kaplan 2005, AJ 129, 2420). NOVAS should deal with all types of astrometric binary systems.

The binary star module that is planned for NOVAS and is the subject of this paper will provide both the absolute and relative positions of the two components of a binary system, at a requested date, given known orbital elements and standard astrometric parameters. Thus, it covers what may called the "forward problem" of producing binary star ephemerides given an orbit already determined. It does not deal with the "backward problem" of determining an orbit from observations. However, once an initial orbit has been determined for a binary system, follow-up observations can improve the orbit through differential correction; and a correct formulation of the forward problem is necessary to avoid systematic errors in the improved orbital elements. That is, the O-C's should be as correct as possible. Given perfect orbital elements, the output of the NOVAS module (the C's) should be accurate to better than 1 mas.

Binary stars are perhaps the purest example of the two-body problem in astronomy, and the computation of the relative positions of the two components in a local space-fixed reference system is non-controversial. However, taking that information and predicting the values of astrometric parameters for an Earth-based observer requires a more sophisticated model of the whole geometry than is usually applied. In particular, at high levels of astrometric accuracy, the 3-D nature of the problem comes into play in surprising ways.

Available Orbital Data for Binary Stars

The data set used is the Sixth Catalog of Orbits of Visual Binary Stars, available on CD from the U.S. Naval Observatory and online at http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/orb6 (it is an update to Hartkopf, Mason, & Worley 2001, AJ 122, 3472).[1] The catalog provides 2518 orbits of 2413 systems, although not all systems have complete sets of elements. The elements given are the standard set of *P, a, e, i, ω, Ω,* and *T*, where *P* is the period, *a* the semimajor axis in angular measure (usually arcseconds), and T the epoch of periastron. The angular elements are expressed in degrees, and *P* and *T* may be in a variety of time units. Uncertainties are also provided. The catalog will be referred to here as ORB6.

The reference system in which the elements are expressed is the "plane of the sky", which is orthogonal to the line of sight. Within this plane, the azimuthal origin is the north direction (position angle zero) specified by an eighth parameter, a date *E*, which represents the "equinox, if any, to which the node [*Ω*] refers". That is, the north direction, at the position of the binary system on the celestial sphere, is toward the north celestial pole of date *E*.

[1] The orbital elements of spectroscopic-only binary systems, for example, those listed in the SB9 catalog, are not suitable for this project since *Ω* is not known, and *a* and *i* are known only in the combination *a sin i*.

Ambiguous Aspects of the Orbital Parameters

Because ORB6 is a collection of orbits determined by different investigators, it is a heterogeneous source of information. Several aspects of the data in ORB6 are not well defined and therefore present an obstacle to precise computations. About 1/3 of the orbits do not have the epoch *E* specified (most of the specified *E*'s are 2000). Orbital elements *P* and *T*, which are times, are often expressed as a year + fraction, in units of Besselian years, but the conversion to more standard time units is not given. In a more subtle issue, it is unclear what line of sight should be used to define the "plane of the sky". Stars move on the celestial sphere (i.e., with respect to the ICRS axes) due to proper motion, aberration, and other effects. Aberration shifts our line of sight to every star, regardless of distance, by 41 arcseconds over the course of every year.

To move forward, I had to make several (arbitrary) assumptions:

★ If *E* is not specified, then *E*=2000, i.e., *E* = J2000.0 = JD 2451545.0 (TT).

★ The conversion from Besselian to Julian epochs and time units is done according to a conventional expression (using fixed-length years) recommended by the IAU in 1976.

★ The plane of the sky for the orbit is orthogonal to the direction toward the A component of the system defined by its *mean place at epoch E*.

★ The direction of the orbital node, i.e., the sign of the inclination, is known (even though in many cases it is ambiguous).

★ Units of arcseconds used for the semimajor axis *a* should be considered to be a measure of length; to convert to astronomical units (au), divide by the parallax of the system.

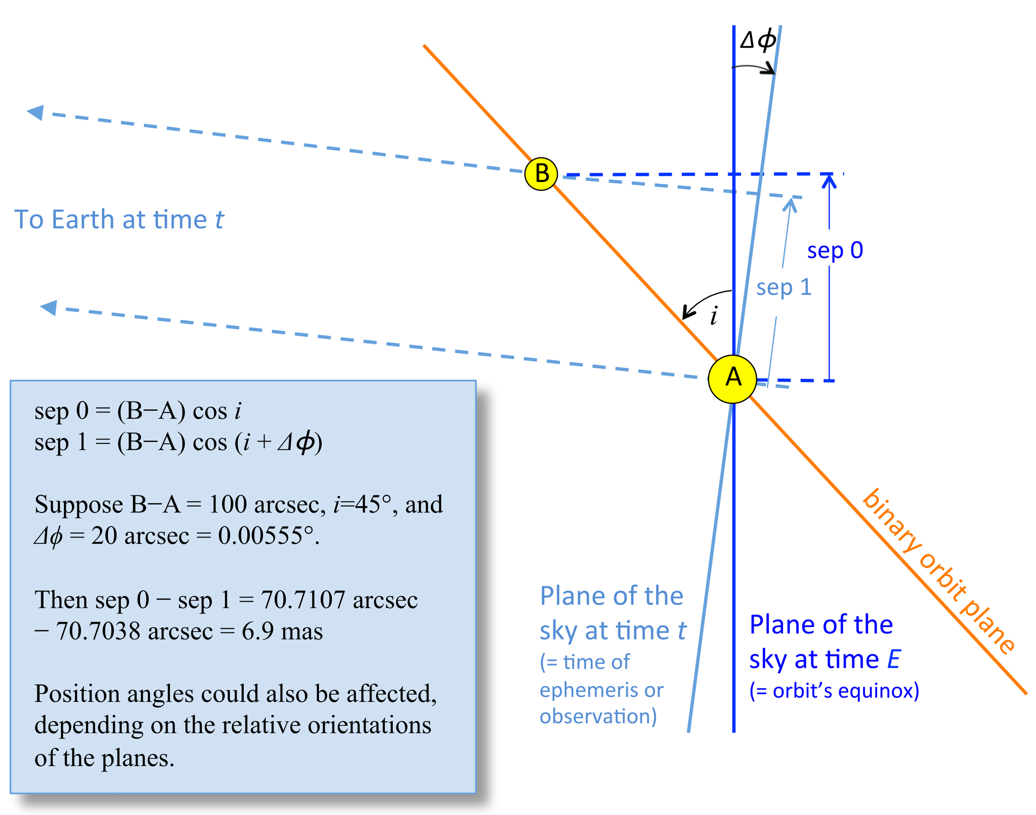
⇨ **It would be quite useful if the appropriate IAU body would establish conventions such as the above for future binary star work.**

Geometry Usually Considered

In computing observables from orbital elements, we can take the solution of the two-body problem for binary stars as a given. Standard texts such as Aitken (1964), *The Binary Stars*, then advise correcting for the change in the north direction, due to precession, between the epoch of the orbit (*E*) and the epoch of observation, and similarly for the change in north direction due to the star's proper motion (although aberration is usually a larger effect). Simple formulas are provided. Some texts also give corrections for change of scale and change of light-time, for those systems with significant radial velocities.

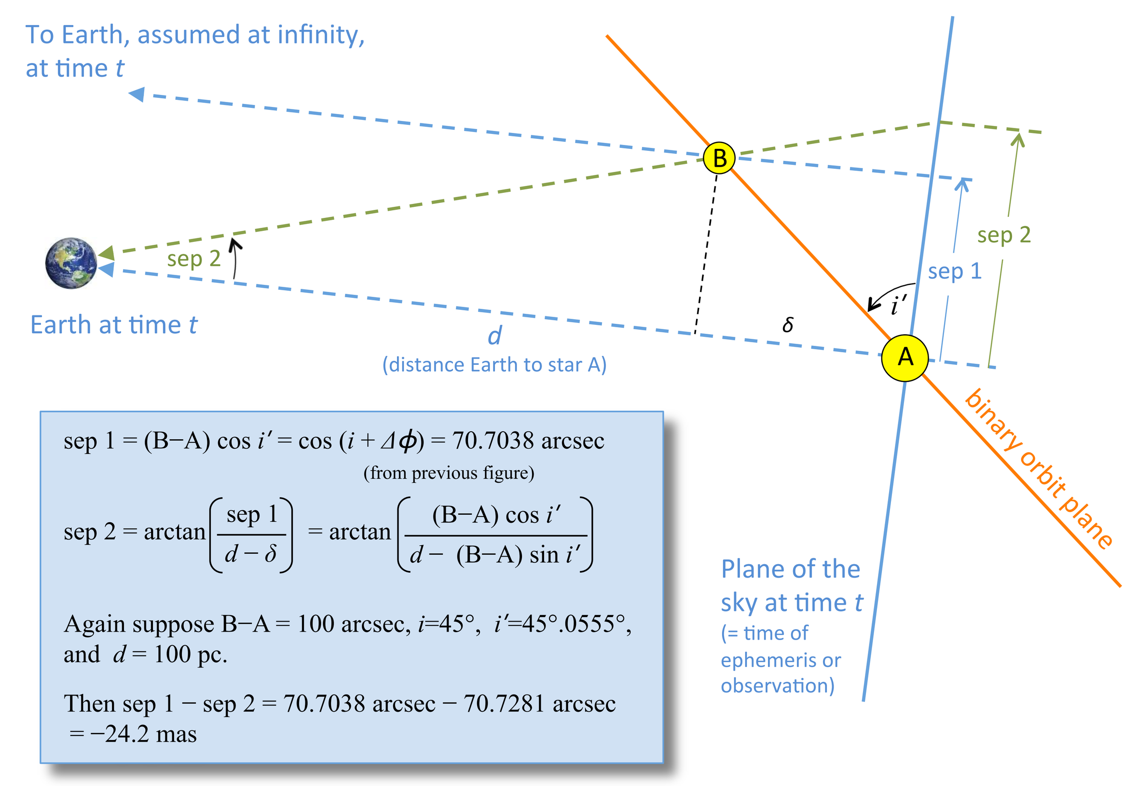
Other Geometry To Consider

In the module developed for NOVAS, the change in north direction is part of a general transformation between two 3-D reference systems that takes into account the changing angle at which we view the binary system. For wide pairs, the component of the stars' separation along the line of sight (z axis) can result in a significant shift of apparent separation when our viewing angle changes, even for the slight change caused by aberration — see figure below.

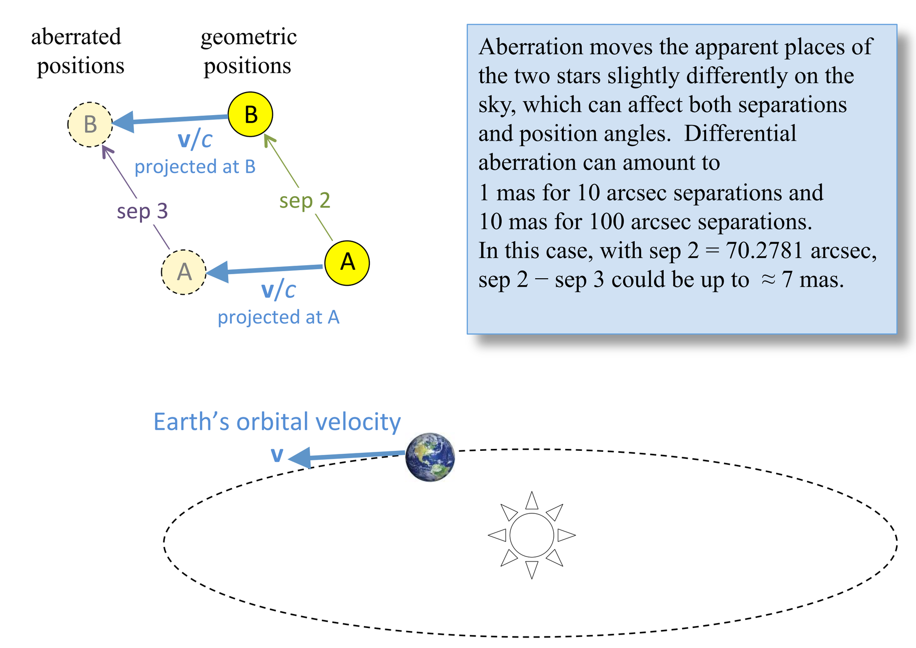


Other Geometry to Consider (cont.)

Another effect arises from the 3-D nature of the relative positions of the two stars. For nearby, wide systems, the fact that B is closer (or farther) than A affects the separation we measure; we should not assume a viewpoint at infinity for these systems.



One more effect to consider: differential aberration between the two components.



NOVAS Algorithm

The algorithm adopted for the NOVAS double star algorithm can be summarized as follows. Some details (such as change of units) are omitted, and for practical reasons the order of computations in the software is somewhat different.

1) Using standard astrometric parameters (e.g., from the Hipparcos catalog) for component A, or the system's center of mass, compute:
   $\mathbf{a}_E$, the mean place direction vector of A at epoch *E* (epoch of orbit),
   $\mathbf{a}_O$, the mean place direction vector of A at the time of observation,
   where both vectors are ss barycentric and with respect to the ICRS axes.

2) Compute the 3x3 transformation matrix between the plane-of-the-sky coordinate system defined by $\mathbf{a}_E$ and the ICRS.

3) Compute the change in scale and light-time between epoch *E* and the time of observation, due to the radial velocity of the system.

4) Using standard Keplerian orbit algorithms, applied to the orbital elements given in ORB6, compute **d**, the 3-vector displacement of component B wrt component A, at the time of observation (corrected for the change in light-time), in the plane-of-the-sky coordinate system defined by $\mathbf{a}_E$.

5) Correct **d** for change of scale, and transform to the ICRS using the matrix from step 2. Call the result $\mathbf{d}'$.

6) Using *q*, the mass ratio (if known; otherwise set *q*=0), compute $\mathbf{a}_O' = \mathbf{a}_O + q/(1+q)\,\mathbf{d}'$ and $\mathbf{b}_O' = \mathbf{a}_O + 1/(1+q)\,\mathbf{d}'$, the mean place direction vectors of components A and B, at the time of observation, in the ICRS.

7) Compute the apparent place direction vectors separately for A and B components, starting with $\mathbf{a}_O'$ and $\mathbf{b}_O'$; apply parallax, gravitational light bending, and aberration. Obtain apparent *α* and *δ* for both components.

8) Compute differential apparent coordinates (position angle *θ* and separation *ρ*) between A and B.